Abstracts

Plenary Speakers

Peter Bates, Michigan State University
Multiphase Solutions to the Vector Allen-Cahn Equation: Crystalline and Other Complex Symmetric Structures

I will present a systematic study of entire symmetric solutions u : R^n --> R^m to the vector Allen-Cahn equation where the nonlinear potential function, W(u), is smooth, invariant under a reflection group in R^n, nonnegative and with a finite number of zeros. After introducing a general notion of equivariance with respect to a homomorphism between reflection groups in R^n and R^m, two abstract results are proved concerning the cases of the reflection group being finite or discrete. These give the existence of equivariant solutions for each case. The approach is variational and based on a mapping property of the parabolic vector Allen-Cahn equation and on a pointwise estimate for vector minimizers. This unifies and generalizes results of many authors over the last decade. Joint work with G.Fusco and P. Syrnellis.

Mariana Haragus, University of Bourgogne Franche-Comté, France
Bifurcation Theories for a Model from Nonlinear Optics

We consider the Lugiato-Lefever equation, which is a nonlinear Schrödinger-type equation with damping, detuning and driving, derived in nonlinear optics by Lugiato and Lefever (1987). While intensively studied in the physics literature, there are relatively few rigorous mathematical studies of this equation. Of particular interest for the physical problem is the dynamical behavior of periodic and localized steady waves. The underlying mathematical questions concern the existence and the stability of these types of waves. In this talk, I'll show how different tools from bifurcation theory can be used in the mathematical analysis of these questions. The focus will be on periodic waves and their stability.

Kevin Zumbrun, Indiana University
A Stable Manifold Theorem for a Class of Degenerate Evolution Equations

We establish a Stable Manifold Theorem, with consequent exponential decay to equilibrium, for a class of degenerate evolution equations $Au'+u=D(u,u)$ with A bounded, self-adjoint, and one-to-one, but not invertible, and D a bounded, symmetric bilinear map. This is related to a number of other scenarios investigated recently for which the associated linearized ODE $Au'+u=0$ is ill-posed with respect to the Cauchy problem. The particular case studied here pertains to the steady Boltzmann equation, yielding exponential decay of (tails of) large-amplitude shock and boundary layers.

Invited Speakers

Benjamin Akers, Air Force Institute of Technology
Modulational Instabilities of Traveling Waves

A perturbative framework for predicting modulational instabilities of traveling waves will be presented. Asymptotics of the spectrum of deep water waves on one- and two-dimensional interfaces are calculated. The role of the instabilities in spectrum's analyticity will be discussed. The potential for asymptotic aided numerical computations will be explored.

John Albert, University of Oklahoma
Concentration Compactness and the Stability of KdV Multisolitons

Multisoliton solutions of the Korteweg-de Vries equation are critical points for certain constrained variational problems in which the functionals involved are conserved quantities for the equation. One way to understand their stability properties is to show that they are actually minimizers, in a strong sense: all minimizing sequences converge strongly to the set of minimizing multisoliton profiles. This can be done using a variant of the concentration compactness method known as the profile decomposition or bubble decomposition. We discuss the analysis involved in the case of 2-soliton solutions.

Andrew Comech, Texas A&M University
Spectral Stability of Small Amplitude Solitary Waves in the Nonlinear Dirac Equation

We study the point spectrum of the linearization at a solitary wave solution to the nonlinear Dirac equation with the scalar type self-interaction (known as the Soler model). We focus on the spectral stability, that is, the absence of eigenvalues with positive real part, in the non-relativistic limit (small amplitude solitary waves). We prove the spectral stability for particular nonlinearities (in particular, for pure power nonlinearities between cubic and quintic in one spatial dimension) and also for the "charge-critical" cases. An important part of the stability analysis is the proof of the absence of bifurcations of nonzero-real-part eigenvalues from the embedded threshold points. Our approach is based on constructing a new family of exact bi-frequency solitary wave solutions in the Soler model, on using this family to determine the multiplicity of exact $\pm 2\omega i$ eigenvalues of the linearized operator, and on the analysis of the behavior of “nonlinear eigenvalues” (characteristic roots of holomorphic operator-valued functions). This is a joint work with Nabile Boussaid (Besançon).

Anna Ghazaryan, Miami University
On the Stability of Planar Fronts

We consider planar fronts in a class of reaction-diffusion systems with the following property: the linearization of the system about the front has no unstable discrete eigenvalues, but its essential spectrum touches the imaginary axis. For perturbations that belong to the intersection of the exponentially weighted space with the original space without a weight, we use a bootstrapping argument to show that initially small perturbations to the front remain bounded in the original norm and decay algebraically in time in the exponentially weighted norm.

Pelin Guven Geredeli, University of Nebraska
Long Time Behavior Properties of Solutions to Evolutionary PDEs

In this talk, we shall discuss our recent research on various classes of linear and nonlinear partial differential equations (PDE's). There will be a focus here on results that deal with long-time behavior of PDE solutions. We will highlight our various approaches to a given problem, including various PDE techniques that are subsequent to our continuous semigroup formulations. In discussing our results, we will feature our work on: (i) our derived global attractor theory for nonlinear parabolic PDE processes, and hyperbolic plate and beam PDE under nonlinear boundary dissipation, (ii) stability properties of various coupled systems of linear PDE of different type- i.e., hyperbolic vs. parabolic.

Mark Hoefer, University of Colorado
Whitham Theory Applied to Modulated Solitary Waves

Whitham modulation theory is a well-known asymptotic technique to describe the slow modulations of nonlinear, periodic, traveling wave solutions of nonlinear dispersive partial differential equations. The resultant first order, quasi-linear Whitham modulation equations can be obtained by averaging conservation laws for the governing partial differential equation over a period of the periodic wave family. This approach has been successfully utilized for the problem of stability of periodic waves and the propagation of dispersive shock waves with many applications. This talk will focus upon the little-studied, singular, solitary wave (zero wavenumber) limit of the Whitham modulation equations. These equations describe a slowly varying mean flow that is decoupled from the solitary wave modulation. Interestingly, the solitary wave modulation is described by an amplitude field that is spatially defined everywhere; solitary wave trajectories are characteristics of the modulation system. Applications include modulated solitary waves and the propagation of solitary waves through other nonlinear waves such as dispersive shock waves and rarefaction waves. Multiple governing partial differential equations and physical examples will be highlighted.

Peter Howard, Texas A&M University
The Maslov Index for Linear Hamiltonian Systems on [0,1] and Applications to Periodic Waves

Working with general linear Hamiltonian systems on [0,1], and with a wide range of self-adjoint boundary conditions, including both separated and coupled, I will discuss a general framework for relating the Maslov index to spectral counts. As an example of the general framework, I will analyze the spectrum of linear operators obtained when Allen-Cahn equations and systems are linearized about stationary periodic solutions.

Stéphane Lafortune, College of Charleston
Stability of Traveling Waves in a Model for a Thin Liquid Film Flow

We consider a model for the flow of a thin liquid film down an inclined plane in the presence of a surfactant. The model is known to possess various families of traveling wave solutions. We use a combination of analytical and numerical methods to study the stability of the traveling waves. We show that for at least some of these waves the spectra of the linearization of the system about them are within the closed left-half complex plane.

Hung Le, University of Missouri
Elliptic Equations with Transmission and Wentzell Boundary Conditions and an Application to Steady Water Waves in the Presence of Wind

In this talk, we present results about the existence and uniqueness of solutions of elliptic equations with transmission and Wentzell boundary conditions. We provide Schauder estimates and existence results in Holder spaces. As an application, we develop an existence theory for small-amplitude two-dimensional traveling waves in an air-water system with surface tension. The water region is assumed to be irrotational and of finite depth, and we permit a general distribution of vorticity in the atmosphere.

Greg Lyng, University of Wyoming
Coordinates for Multidimensional Evans-Function Computations

The Evans function has become a standard tool in the mathematical study of the stability of nonlinear waves. In particular, computation of its zero set gives a convenient numerical method for determining the point spectrum of the associated linear operator (and thus the spectral stability of the wave in question). In this talk, I will describe the central (and perhaps unexpected) role that coordinate choices play in making Evans-function computations for multidimensional viscous shock waves viable. This is joint work with Blake Barker, Jeff Humpherys, and Kevin Zumbrun.

Yulia Meshkova, St. Petersburg State University
Operator Error Estimates for Homogenization of Periodic Hyperbolic Systems

The talk is devoted to homogenization for solutions of periodic hyperbolic systems with rapidly oscillating coefficients. We wish to approximate solutions in the L_2- and H^1-norms. To obtain estimate in the energy norm we assume that the initial data for the solution is zero. But the initial data for the time derivative of the solution is non-zero. So, the solution can be represented as the corresponding operator sine acting on the non-zero initial data. We obtain principal term of approximation for the solution and approximation in the Sobolev class H^1 with the correction term taken into account. The results can be written as approximations of the operator sine in the uniform operator topology with the precise order error estimates. We use the spectral approach to homogenization problems developed by M. Sh. Birman and T. A. Suslina. The method is based on the scaling transformation, the Floquet-Bloch theory and analytic perturbation theory. It turns out that homogenization is a spectral threshold effect at the bottom of the spectrum. More details: arXiv:1705.02531. The research was supported by project of Russian Science Foundation no. 17-11-01069.

Peter Miller, University of Michigan
Extreme Superposition: Rogue Waves of Infinite Order

Using a recently-obtained analytical representation of the rogue-wave solutions of the focusing nonlinear Schroedinger equation on a nonzero background, we study the asymptotic behavior of rogue waves of increasingly high order. We identify a new solution of the focusing nonlinear Schroedinger equation that we call the rogue wave of infinite order, and show that it approximates high-order rogue waves near the amplitude peak, under suitable rescaling. The rogue wave of infinite order also satisfies ordinary differential equations in space and time related to the Painleve-III hierarchy, and has nontrivial asymptotics with high oscillation and algebraic decay for large x and t. This is joint work with Deniz Bilman and Liming Ling.

Yannan Shen, California State University, Northridge
Nonlinear Waves in Lattices

In this talk we will give some examples of nonlinear waves in lattice dynamical systems, including nonlinear optics, nonlinear metamaterials and granular crystals. We will show we derived from an intrinsically discrete physical system, through a long-wavelength oscillations to a continuous model that can support solitary waves. We will discuss possible connections between continuum and anti-continuum limit and possible future directions of research.

Poster Presentations

Md Al Masum Bhuiyan, University of Texas, El Paso
A Dynamical Analysis of Earthquake Waves by Using Ornstein Uhlenbeck Type Model

This work is devoted to modeling of earthquake wave time series. We propose a stochastic differential equation arising on the superposition of independent Ornstein-Uhlenbeck processes driven by a $\Gamma(m,n)$ process. Superposition of independent $\Gamma(m, n)$ Ornstein-Uhlenbeck processes offers analytic flexibility and provides a dynamic class of continuous time processes capable of exhibiting long memory behavior. The stochastic differential equation is applied to the study of earthquake waves by fitting the superposed $\Gamma(m, n)$ Ornstein-Uhlenbeck model to earthquakes sequences in Arizona, USA. We obtained very good fitting of the observed velocity proportional of earthquakes with the stochastic differential equations, which supports the use of this methodology for the study of non-linear sequences.

Jason Bramburger, Brown University
Snaking in the Swift-Hohenberg Equation in Dimension 1+Epsilon

The Swift-Hohenberg equation is a widely studied partial differential equation which is known to support a variety of spatially localized structures. The one-dimensional equation exhibits spatially localized steady-state solutions which give way to a bifurcation structure known as snaking. That is, these solutions bounce between two different values of the bifurcation parameter while ascending in norm. The mechanism that drives snaking in one spatial dimension is now well-understood, but recent numerical investigations indicate that upon moving to two spatial dimensions, the related radially-symmetric spatially-localized solutions take on a significantly different snaking structure which consists of three major components. To understand this transition we apply a dimensional perturbation in an effort to use well-developed methods of perturbation theory and dynamical systems. In particular, we are able to identify key characteristics that lead to the segmentation of the snaking branch and therefore provide insight into how the bifurcation structure changes with the spatial dimension.

Hong Cai, Brown University
Travelling Front

We think about wave fronts in Rosenzweig-MacArthur system in two cases. One is prey diffuses at the rate much smaller than that of the predator, the other is both of them diffuse slowly.

Paula Egging, University of Nebraska
Uniform Decay of a Structural Acoustic Dynamics

This poster presents recently derived results of uniform rational decay for strong solutions of a canonical structural acoustic PDE which has previously appeared in the literature. Our stability proof depends upon an appropriate invocation of a now well-known resolvent criterion of A. Borichev and Y. Tomilov.

Fazel Hadadifard, University of Kansas
Optimal Time Decay Rates for the Generalized Surface Quasi-Geostrophic Equation

We are interested in finding the optimal decay rate for a generalized fractional surface quasi-geostrophic equation. This type of equation arises frequently in fluid dynamics. To achieve this goal we use a method called the “Scaling Variables”.

Laszlo Kindrat, University of New Hampshire
Numerical Spectral Analysis of the Euler-Bernoulli Beam Model with Non-conservative Boundary Control

Numerical spectral analysis to investigate the vibrational frequencies of a flexible beam model equipped with fully non-conservative linear feedback boundary conditions is presented. The Chebyshev collocation method is used and a novel technique is developed to handle the dynamic boundary conditions. The accuracy of the numerical scheme is demonstrated by showing the rate of convergence of the scheme and by comparing numerical and analytic results. The very good agreement between the asymptotic and numerical approximations of the vibrational spectrum is discussed.

Ang Li, Brown University
Linear Stability of Planar Spiral Waves

Planar spiral waves have been observed in many natural systems and also as solutions of reaction-diffusion equation. Our goal is to investigate the linear stability of spectrally stable spiral waves by establishing pointwise estimate of the associated Green's function. The essential spectrum of the linearization about a spiral wave has countably many branches that touch the imaginary axis and is therefore not sectorial: we plan to use the Gearhart-Prüss Theorem to prove the spectral mapping theorem for the linearization about a spiral wave. Afterwards, Laplace-transform techniques will be used to derive the pointwise estimates of the Green's function.

Satbir Malhi, University of Kansas
Energy Decay Rates for the Fractional Klein-Gordon Equation with Periodic Damping

In this poster, we consider the Fractional Damped Klein-Gordon equation with spatial fractional derivative of order s. We prove that the energy of the solution for the equation decays at a polynomial rate intern of s for 0< s<2 and at exponential rate when s >2, provided damping coefficient is non-trivial and periodic, or more generally strictly positive on a periodic set. The approach we use in this paper is based on the asymptotic theory of C0 semigroups in which one can relate the decay rate of energy and the resolvent growth of the semigroup generator. An important ingredient of the proof is to derive the observability estimate for the fractional Laplacain which has potential applications in control theory.

Ross Parker, Brown University
Stability of Double Pulse Solutions to the 5th order KdV Equation, a Numerical Approach

The fifth-order Korteweg-de Vries equation (KdV5) is a nonlinear partial differential equation used to model dispersive phenomena such as plasma waves and capillary-gravity water waves. For wave speeds exceeding a critical threshold, KdV5 admits a countable family of double-pulse traveling-wave solutions, where the two pulses are separated by a phase parameter multiplied by an integer N. It is known that the double pulses are unstable for even N, and it has also been shown that these pulses have either a quadruplet of eigenvalues or a pair of purely imaginary eigenvalues near the origin when N is odd. Moreover, the latter case arises provided the associated eigenfunctions are square-integrable. It is not known which of these two cases arises in KdV5. We provide the results of extensive numerical computations that indicate that the eigenvalue are indeed purely imaginary. We also present numerical Krein matrix computation for periodic waves that indicate that long-wavelength double-pulse wave trains are stable.

Iurii Posukhovskyi, University of Kansas
On the Normalized Ground States for the Kawahara Equation and a Fourth Order NLS

We consider the Kawahara model and two fourth order semi-linear Schrödinger equations in any spatial dimension. We construct the corresponding normalized ground states, which we rigorously show to be spectrally stable. For the Kawahara model, our results provide a significant extension in parameter space of the current rigorous results. At the same time, we verify and clarify recent numerical simulations of the stability of these solitons. For the fourth order NLS models, we improve upon recent results on stability of very special, explicit solutions in the one-dimensional case. Our multidimensional results for fourth order NLS seem to be the first of its kind. Of particular interest is a new paradigm that we discover herein. Namely, all else being equal, the form of the second order derivatives (mixed second derivatives vs. pure

Laplacian) has implications on the range of existence and stability of the normalized waves.

Abba Ramadan, University of Kansas
Existense of Traveling Waves for a Class of Nonlocal Nonlinear Equations with Bell Shaped Kernels

This work is to investigate the existence of traveling wave solutions of a general class of nonlocal wave equations: $u_{tt} - a^2 u_{xx} = (\beta * u^p)_{xx}$, $p > 1$. Members of the class arise as mathematical models for the propagation of waves in a wide variety of situations. We assume that the kernel β is a bell-shaped function satisfying some mild differentiability and growth conditions. Taking advantage of growth properties of bell shaped functions, we give a proof for the existence of bell-shaped traveling wave solutions.

Connor Smith, University of Kansas
Metastable Traveling Fronts Arising in Nanoscale Pattern Formation

We study an equation that models ripple formation when a flat surface is bombarded by an ion beam. The experimentally observed ripples consist of parts with a certain positive slope, parts with a certain negative slope, and transitions between the two slopes. The transitions admit a solitary wave solution with stable point spectrum but unstable essential spectrum. At first glance this may be written off as “unstable.” However numerical results suggest that the instability has two parts; a convecting part that saturates to some value and an exponentially decaying part that slightly modulates the underlying transition. We define an exponential weight that decays in the direction the first part of the instability travels in, allowing us to focus on the second part of the instability. In this exponentially weighted space we obtain a linear stability result. Inspired by the “repeating” part of the experimental results, we also consider “gluing” together solitary waves in an ad hoc periodic pattern. For some specific methods of gluing the entire spectrum is stable, with the implication that while an individual transition is unstable, the instability is benign enough that it can be stabilized by repeating the transition.

Patrick Sprenger, University of Colorado
The Step Initial Value Problem for a Fifth Order KdV Equation

We consider a step initial value problem for a fifth order of equation of KdV type where the third order conservative derivative is replaced with a fifth order spatial derivative. The discontinuity is resolved via an unsteady, modulated nonlinear wavetrain consisting of two distinct regions. The first region consists of a equilibrium with a heteroclitic connection to a homoclinic periodic orbit that seems to tend to a genuine traveling wave solution to the equation. The finite amplitude wavetrain is terminated by partial dispersive shock wave decaying to zero at the right. Whitham modulation theory is developed in the weakly nonlinear limit so that the properties of the partial dispersive shock wave can be quantified.

Selim Sukhtaiev, Rice University
Anderson Localization for Radial Tree Graphs With Random Branching Numbers

We prove Anderson localization for the discrete Laplace operator on radial tree graphs with random branching numbers. Our method relies on the representation of the Laplace operator as the direct sum of half-line Jacobi matrices whose entries are non-degenerate, independent, identically distributed random variables with singular distributions.

Qingtian Zhang, University of California, Davis
Uniqueness of Conservative Solution to Camassa-Holm and Variational Wave Equations

Camassa-Holm equation and variational wave equation are two models from shallow water waves and nematic liquid crystals separately. In this poster, I will provide the proof of the uniqueness of the conservative weak solutions for these two equations.